MATHEMATICS

Each question carries 2 mark

Full Marks : 100

1. Let $A = \lim_{x \to 1} \sqrt{x^4 - 4x^3 + 5x^2 - 2x}$ and $B = \lim_{x \to -3} \frac{x+1}{(x+3)^2}$. Then

(A) A does not exist and B = $-\infty$

(B) A exists and equals to 0 , and B = $-\infty$

(C) A exists and equals to 0, and B = $+\infty$

(D) Both A and B do not exist.

2. If the k-th term t_k of a series is given by $t_k = \frac{k}{1+k^2+k^4}$, then the value of $\lim_{n \to \infty} \sum_{k=1}^{n} t_k$ is

(A) 1

(B) ∞

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

3. Given $f(\theta) = \begin{cases} (\text{Cos}\theta - \text{Sin}\theta)^{\text{cosec}\,\theta}, & \text{if} -\frac{\pi}{2} < \theta < 0 \\ a, & \text{if } \theta = 0 \\ \frac{e^{1/\theta} + e^{2/\theta} + e^{3/\theta}}{ae^{2/\theta} + be^{3/\theta}}, & \text{if } 0 < \theta < \frac{\pi}{2} \end{cases}$

if $f(\theta)$ is continuous at $\theta = 0$, then

(A) a = e, b = e

(B) $a = \frac{1}{e}$, b = e

(C) a = e, $b = \frac{1}{e}$

(D) $a = \frac{1}{e}$, $b = \frac{1}{e}$

4. Let f(x) = [n + p sin x], $x \in (0, \pi)$, n is an integer and p is a prime number, where [.] denotes the greatest integer function. Then the number of points where f(x) is not differentiable, is

(A) p

(B) p−1

(C) 2p

(D) 2p−1

5. Let f(x) be a real function not identically zero such that $f(x+y^{2n+1}) = f(x) + f(y)^{2n+1}$, $n \in$ ▯ and x, y are any real number and $f'(0) \geq 0$. The values of f(5) and $f'(10)$ are respectively :

(A) 25 and 100

(B) 5 and 1

(C) 4 and 40

(D) 10 and 5

6. Find the points for maxima and minima of the function

$$f(x) = \int_1^x \left[2(t-1)(t-2)^3 + 3(t-1)^2(t-2)^2\right]dt$$

(A) maximum at $x = 1$ and minimum at $x = \frac{7}{5}$

(B) maximum at $x = \frac{7}{5}$ and minimum at $x = 2$

(C) maximum at $x = 2$ and minimum at $x = \frac{7}{5}$

(D) maximum at $x = 2$ and minimum at $x = 1$

7. If $y = m\sin(m\sin^{-1}x)$, then the value of $(x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is

(A) $-m^2y$ (B) $-my$

(C) m^2y (D) my

8. If $y = \sin^{-1}x = a_0 + a_1x + a_2x^2 + \ldots$, then $n(n+1)a_{n+1}$ equals

(A) $(n-1)^2a_{n-1}$ (B) n^2a_n

(C) n^2a_{n+1} (D) $(n+1)^2a_{n+1}$

9. The Maclaurin's series of the function $f(x) = \log_e\left(x + \sqrt{1+x^2}\right)$ is

(A) $x - \frac{x^3}{3!}1^2 + \frac{x^5}{5!}(3^2.1^2) - \frac{x^7}{7!}(5^2.3^2.1^2) + \ldots$

(B) $x + \frac{x^3}{3!}1^2 + \frac{x^5}{5!}(3^2.1^2) + \frac{x^7}{7!}(5^2.3^2.1^2) + \ldots$

(C) $x - \frac{x^2}{2!}1^2 + \frac{x^4}{4!}(2^2.1^2) - \frac{x^6}{6!}(4^2.2^2.1^2) + \ldots$

(D) $x + \frac{x^2}{2!}1^2 + \frac{x^4}{4!}(2^2.1^2) + \frac{x^6}{6!}(4^2.2^2.1^2) + \ldots$

10. Let [x] denotes the greatest integer function of x. The value of α for which the function

$$f(x)=\begin{cases}\dfrac{\sin[-x^2]}{[-x^2]}, & x\neq 0\\ \alpha, & x=0\end{cases}$$ is continuous at x = 0, is

(A) 0 (B) sin(−1)

(C) sin(1) (D) 1

11. The smallest integral value of *k*, for which both the roots of the equation $x^2-8kx+16(k^2-k+1)=0$ are real, distinct and have values at least 4, is

(A) 1 (B) 2

(C) 3 (D) 4

12. If the function $f(x)=x^3+3^{x/2}$ and $g(x)=f^{-1}(x)$, then the value of $g'(1)$ is

(A) 1 (B) 2

(C) 0 (D) 3

13. Let P(6,3) be a point on the hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$. If the normal at the point P intersects the x-axis at (9,0), then the eccentricity of the hyperbola is

(A) $\sqrt{\dfrac{5}{2}}$ (B) $\sqrt{\dfrac{3}{2}}$

(C) $\sqrt{2}$ (D) $\sqrt{3}$

14. The circle passing through the point (-1,0) and touching the y-axis at (0,2) also passes through the point

(A) $\left(-\dfrac{3}{2},0\right)$ (B) $\left(-\dfrac{5}{2},2\right)$

(C) $\left(-\dfrac{3}{2},\dfrac{5}{2}\right)$ (D) $(-4,0)$

15. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i} - \hat{j}$, $4\hat{i}$, $3\hat{i} + 3\hat{j}$ and $-3\hat{i} + 2\hat{j}$ respectively. The quadrilateral PQRS must be a

(A) Parallelogram, which is neither a rhombus nor a rectangle
(B) Square
(C) Rectangle, but not a square
(D) Rhombus, but not a square.

16. If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then $|d|$ is

(A) 4
(B) 6
(C) 8
(D) 10

17. Let p be an odd prime number and T_p be the following set of 2 x 2 matrices :

$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, \ldots. p-1\} \right\}$. The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det(A) is divisible by p is

(A) $(p-1)^2$
(B) $2(p-1)$
(C) $(p-1)^2 + 1$
(D) $2p-1$

18. Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

(A) $x + 2y - 2z = 0$
(B) $3x + 2y - 2z = 0$
(C) $x - 2y + z = 0$
(D) $5x + 2y - 4z = 0$

19. The length of the chord intercepted by the parabola $y^2 = 8x$ on the straight line $2x - y - 3 = 0$ is

(A) $2\sqrt{5}$ (B) $3\sqrt{5}$

(C) $4\sqrt{5}$ (D) $5\sqrt{5}$

20. The co-ordinate of the middle point of the chord of the hyperbola $x^2 - 4y^2 = 9$ along the straight line $x + 4y + 3 = 0$ is

(A) $(1, -1)$ (B) $(-1, 1)$

(C) $(0, 1)$ (D) $(-1, 0)$

21. Let f, g and h be real-valued functions defined on the interval [0,1] by, $f(x) = e^{x^2} + e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c denotes, respectively, the absolute maxima of f, g and h on [0, 1], then

(A) $a = b$ and $c \neq b$ (B) $a = c$ and $a \neq b$

(C) $a \neq b$ and $c \neq b$ (D) $a = b = c$

22. Let p and q be real numbers such that $p \neq 0, p^3 \neq q$, and $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

(A) $(P^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$ (B) $(P^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

(C) $(P^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$ (D) $(P^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

23. In an equilateral triangle, 3 coins of radii 1 unit each are kept so that they touch each other and also the sides of the triangle. Area of the triangle is

(A) $4 + 2\sqrt{3}$ (B) $6 + 4\sqrt{3}$

(C) $12 + \frac{7\sqrt{3}}{4}$ (D) $3 + \frac{7\sqrt{3}}{4}$

24. If P(x) is a polynomial of degree less than or equal to 2 and S is the set of all such polynomials such that $P(1) = 1$, $P(0) = 0$, and $P'(x) > 0 \ \forall \ x \in [0, 1]$, then

(A) $S = \Phi$

(B) $S = \{(1-a)x^2 + ax, 0 < a < 2\}$

(C) $S = \{(1-a)x^2 + ax, a \in (0, \infty)\}$

(D) $S = \{(1-a)x^2 + ax, 0 < a < 1\}$

25. The value of $\int_0^1 \frac{x^4(1-x)^4}{1+x} dx$ is

(A) $\frac{22}{7} - \pi$

(B) $\frac{2}{105}$

(C) 0

(D) $\frac{71}{15} - \frac{3\pi}{2}$

26. The value of $\int_0^1 \sqrt{\frac{1-x}{1+x}} \, dx$ is

(A) $\frac{\pi}{2} + 1$

(B) $\frac{\pi}{2} - 1$

(C) -1

(D) 1

27. The value of $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ is

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) π

(D) 0

28. If f(x) is differentiable and $\int_0^{t^2} x f(x)\,dx = \frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals

(A) 2/5 (B) −5/2

(C) 1 (D) 5/2

29. If $u = \log\left(\frac{x^4 + y^4}{x + y}\right)$, then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is

(A) 1 (B) 2

(C) 3 (D) 4

30. The integrating factor of the differential equation $(2x^2y^2 + y)dx + (-x^3y + 3x)dy = 0$ is

(A) $x^{-11/7}$ (B) $y^{-19/7}$

(C) $x^{-11/7}y^{-19/7}$ (D) None of the above

31. The differential equation $x\frac{dy}{dx} - y + 1 = 0$ and $y(0) = 1$ has

(A) No solution (B) exactly one solution

(C) at most one solution (D) more than one solution

32. The solution of the differential equation $\frac{dy}{dx} + yx = y^2e^{x^2/2}\sin x$ is

(A) $y(\cos x + c)e^{x^2/2} = 1$ (B) $x(\cos y + c)e^{x^2/2} = 1$

(C) $y(\sin x + c)e^{x^2/2} = 1$ (D) $x(\sin y + c)e^{x^2/2} = 1$

33. If the position vectors of A, B, C are $2\hat{i} + 3\hat{j} - \hat{k}$, $4\hat{i} + 5\hat{j} + \hat{k}$ and $3\hat{i} + 6\hat{j} - 3\hat{k}$ respectively, then Δ ABC is

(A) equilateral (B) equiangular

(C) right-angled (D) none of the above

34. If the vectors $x\hat{i}-3\hat{j}+7\hat{k}$ and $\hat{i}+y\hat{j}-z\hat{k}$ are collinear, then the value of $\frac{xy^2}{z}$ equals

(A) 9/7 (B) −9/7

(C) 6/7 (D) −6/7

35. Let $\vec{u}=\hat{i}+\hat{j}$, $\vec{v}=\hat{i}-\hat{j}$ and $\vec{w}=\hat{i}+2\hat{j}+3\hat{k}$. If $\vec{n}$ is a unit vector such that $\vec{u}\cdot\vec{n}=0$ and $\vec{v}\cdot\vec{n}=0$ then $|\vec{w}\cdot\vec{n}|$ equals

(A) 0 (B) 1

(C) 2 (D) 3

36. If the system of equations

$x-ky-z=0$

$kx-y-z=0$

$x+y-z=0$

has a non-zero solution, then the possible values of k are

(A) −1, 2 (B) 1, 2

(C) 0, 1 (D) −1, 1

37. The value of 'a' for which the system of equations

$x+ay=0$

$y+az=0$

$z+ax=0$

has infinitely many solutions is

(A) 1 (B) 0

(C) −1 (D) None of the above

38. If the system of equations

$x=a(y+z)$

$y=b(z+x)$

$z=c(x+y)$, (a, b, c, $\neq -1$)

has a non-zero solution, then the value of $\frac{a}{1+a}+\frac{b}{1+b}+\frac{c}{1+c}$ is

(A) 2 (B) 1

(C) 0 (D) −1

39. The minimum value of $3x + 2y$ when x, y are positive variables subject to the condition $x^2y^3 = 48$ is :

(A) 10
(B) 20
(C) 30
(D) 40

40. Let $f(x) = \begin{cases} 0, -1 \le x \le 0, \\ x^4, 0 < x \le 1. \end{cases}$ If $f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!} x^k + \frac{f^{(n+1)}(\xi)}{(n+1)!} x^{n+1}$ is the Taylor's formula for f about x = 0 with minimum possible value of n, then the value of ξ for $0 < x \le 1$ is

(A) $\frac{x}{2}$
(B) $\frac{x}{4}$
(C) $\frac{x}{16}$
(D) $\frac{x}{32}$

41. In a certain city the daily consumption of electric power in million of KW hours is a random variable X having a gamma distribution $f_x(x) = \begin{cases} \frac{1}{16} e^{-\frac{x}{2}} x^2, & x > 0 \\ 0, & x < 0 \end{cases}$

The probability that on any given day, the daily power consumption will exceed 12 million KW hours is

(A) 0.012
(B) 0.03
(C) 0.06
(D) 0.015

42. A radioactive source emits on the average 2.5 particles per second. The probability that 2 or more particles will be emitted in an interval of 4 seconds is

(A) $1 - e^{-10}$
(B) $1 - 10e^{-10}$
(C) $11\, e^{-10}$
(D) $1 - 11e^{-10}$

43. 'a' is a fixed point in the interval (0,1). A random variable X is uniformly distributed in that interval. The correlation coefficient between the random variable X and the distance Y from the point 'a' to X is

(A) $\frac{2a^3 - 3a + 2}{6}$ (B) $\frac{a^3 - 2a + 2}{6}$

(C) $\frac{4a^3 - 6a^2 + 1}{12}$ (D) $\frac{2a^3 - 3a + 2}{12}$

44. Let X denote the number of accidents in a factory per weak having p.m.f

$P_x(x) = \frac{k}{(x+1)(x+2)}$, $x = 0, 1, 2, \ldots$. The value of k is

(A) 2 (B) $\frac{3}{2}$

(C) 1 (D) None of the above

45. If X is standard normal variate, then $Y = \frac{X^2}{2}$ is

(A) γ (1) Variate (B) $\chi^2\left(\frac{1}{2}\right)$ variate

(C) $\gamma\left(\frac{1}{2}\right)$ variate (D) None of the above

46. The determinant $\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix} = 0$ if

(A) x, y, z are in A. P. (B) x, y, z are in G. P.

(C) x, y, z are in H. P. (D) xy, yz, zx are in A. P.

47. If ω is a cube root of unity, then a root of the equation $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0$ is

(A) $x = 1$ (B) $x = \omega$

(C) $x = \omega^2$ (D) $x = 0$

48. Let X be a Poisson random variable with parameter λ and $P(X=0) = P(X=1)$. Then $(P(X>1)$ equals

(A) $1-e^{-\lambda}$

(B) $1-\lambda e^{-\lambda}$

(C) $1-e^{-\lambda}-\lambda e^{-\lambda}$

(D) $e^{-\lambda}+\lambda e^{-\lambda}$

49. If $a^2+b^2+c^2=1$, then $ab+bc+ca$ lies in the interval

(A) $[1/2, 2]$

(B) $[-1, 2]$

(C) $[-1/2, 1]$

(D) $[-1, 1/2]$

50. Value of $\sqrt{x^2+x}+\dfrac{\tan^2\alpha}{\sqrt{x^2+x}}$, for $x>0$, and $\alpha \in \left(0, \dfrac{\pi}{2}\right)$ is always greater than or equal to

(A) 2

(B) $\dfrac{5}{2}$

(C) $2\tan\alpha$

(D) $\sec\alpha$